Image Reconstruction From Projections

This book introduces the classical and modern image reconstruction technologies. It covers topics in two-dimensional (2D) parallel-beam and fan-beam imaging, three-dimensional (3D) parallel ray, parallel plane, and cone-beam imaging. Both analytical and iterative methods are presented. The applications in X-ray CT, SPECT (single photon emission computed tomography), PET (positron emission tomography), and MRI (magnetic resonance imaging) are discussed. Contemporary research results in exact region-of-interest (ROI) reconstruction with truncated projections, Katsevich's cone-beam filtered backprojection algorithm, and reconstruction with highly under-sampled data are included. The last chapter of the book is devoted to the techniques of using a fast analytical algorithm to reconstruct an image that is equivalent to an iterative reconstruction. These techniques are the author's most recent research results. This book is intended for students, engineers, and researchers who are interested in medical image reconstruction. Written in a non-mathematical way, this book provides an easy access to modern mathematical methods in medical imaging. Table of Content: Chapter 1 Basic Principles of Tomography 1.1 Tomography 1.2 Projection 1.3 Image Reconstruction 1.4 Backprojection 1.5 Mathematical Expressions Problems References Chapter 2 Parallel-Beam Image Reconstruction 2.1 Fourier Transform 2.2 Central Slice Theorem 2.3 Reconstruction Algorithms 2.4 A Computer Simulation 2.5 ROI Reconstruction with Truncated Projections 2.6 Mathematical Expressions (The Fourier Transform and Convolution , The Hilbert Transform and the Finite Hilbert Transform , Proof of the Central Slice Theorem, Derivation of the Filtered Backprojection Algorithm , Expression of the Convolution Backprojection Algorithm, Expression of the Radon Inversion Formula ,Derivation of the Backprojection-then-Filtering Algorithm Problems References Chapter 3 Fan-Beam Image Reconstruction 3.1 Fan-Beam Geometry and Point Spread Function 3.2 Parallel-Beam to Fan-Beam Algorithm Conversion 3.3 Short Scan 3.4 Mathematical Expressions (Derivation of a Filtered Backprojection Fan-Beam Algorithm, A Fan-Beam Algorithm Using the Derivative and the Hilbert Transform) Problems References Chapter 4 Transmission and Emission Tomography 4.1 X-Ray Computed Tomography 4.2 Positron Emission Tomography and Single Photon Emission Computed Tomography 4.3 Attenuation Correction for Emission Tomography 4.4 Mathematical Expressions Problems References Chapter 5 3D Image Reconstruction 5.1 Parallel Line-Integral Data 5.2 Parallel Plane-Integral Data 5.3 Cone-Beam Data (Feldkamp's Algorithm, Grangeat's Algorithm, Katsevich's Algorithm) 5.4 Mathematical Expressions (Backprojection-then-Filtering for Parallel Line-Integral Data, Filtered Backprojection Algorithm for Parallel Line-Integral Data, 3D Radon Inversion Formula, 3D Backprojection-then-Filtering Algorithm for Radon Data, Feldkamp's Algorithm, Tuy's Relationship, Grangeat's Relationship, Katsevich's Algorithm) Problems References Chapter 6 Iterative Reconstruction 6.1 Solving a System of Linear Equations 6.2 Algebraic Reconstruction Technique 6.3 Gradient Descent Algorithms 6.4 Maximum-Likelihood Expectation-Maximization Algorithms 6.5 Ordered-Subset Expectation-Maximization Algorithm 6.6 Noise Handling (Analytical Methods, Iterative Methods, Iterative Methods) 6.7 Noise Modeling as a Likelihood Function 6.8 Including Prior Knowledge 6.9 Mathematical Expressions (ART, Conjugate Gradient Algorithm, ML-EM, OS-EM, Green's One-Step Late Algorithm, Matched and Unmatched Projector/Backprojector Pairs) 6.10 Reconstruction Using Highly Undersampled Data with l0 Minimization Problems References Chapter 7 MRI Reconstruction 7.1 The 'M' 7.2 The 'R' 7.3 The 'I'; (To Obtain z-Information, x-Information, y-Information) 7.4 Mathematical Expressions Problems References Indexing

"Medical Image Reconstruction: A Conceptual Tutorial" introduces the classical and modern image reconstruction technologies, such as two-dimensional (2D) parallel-beam and fan-beam imaging, three-dimensional (3D) parallel ray, parallel plane, and cone-beam imaging. This book presents both analytical and iterative methods of these technologies and their applications in X-ray CT (computed tomography), SPECT (single photon emission computed tomography), PET (positron emission tomography), and MRI (magnetic resonance imaging). Contemporary research results in exact region-of-interest (ROI) reconstruction with truncated projections, Katsevich's cone-beam filtered backprojection algorithm, and reconstruction with highly undersampled data with l0-minimization are also included. This book is written for engineers and researchers in the field of biomedical engineering specializing in medical imaging and image processing with image reconstruction. Gengsheng Lawrence Zeng is an expert in the development of medical image reconstruction algorithms and is a professor at the Department of Radiology, University of Utah, Salt Lake City, Utah, USA.

Focusing on mathematical methods in computer tomography, Image Processing: Tensor Transform and Discrete Tomography with MATLAB® introduces novel approaches to help in solving the problem of image reconstruction on the Cartesian lattice. Specifically, it discusses methods of image processing along parallel rays to more quickly and accurately reconstruct images from a finite number of projections, thereby avoiding overradiation of the body during a computed tomography (CT) scan. The book presents several new ideas, concepts, and methods, many of which have not been published elsewhere. New concepts include methods of transferring the geometry of rays from the plane to the Cartesian lattice, the point map of projections, the particle and its field function, and the statistical model of averaging. The authors supply numerous examples, MATLAB®-based programs, end-of-chapter problems, and experimental results of implementation. The main approach for image reconstruction proposed by the authors differs from existing methods of back-projection, iterative reconstruction, and Fourier and Radon filtering. In this book, the authors explain how to process each projection by a system of linear equations, or linear convolutions, to calculate the corresponding part of the 2-D tensor or paired transform of the discrete image. They then describe how to calculate the inverse transform to obtain the reconstruction. The proposed models for image reconstruction from projections are simple and result in more accurate reconstructions. Introducing a new theory and methods of image reconstruction, this book provides a solid grounding for those interested in further research and in obtaining new results. It encourages readers to develop effective applications of these methods in CT.

Image Reconstruction from ProjectionsThe Fundamentals of Computerized Tomography

This graduate textbook explains image geometry, and elaborates on image enhancement in spatial and frequency domain, unconstrained and constrained restoration and restoration from projection, and discusses various coding technologies such as predictive coding and transform coding. Rich in examples and exercises, it prepares electrical engineering and computer science students for further studies on image analysis and understanding.

Computed Tomography gives a detailed overview of various aspects of computed tomography. It discusses X-ray CT tomography from a historical point of view, the design and physical operating principles of

computed tomography apparatus, the algorithms of image reconstruction and the quality assessment criteria of tomography scanners. Algorithms of image reconstruction from projections, a crucial problem in medical imaging, are considered in depth. The author gives descriptions of the reconstruction methods related to tomography scanners with a parallel X-ray beam, trough solutions with fan-shaped beam and successive modifications of spiral scanners. Computed Tomography contains a dedicated chapter for those readers who are interested in computer simulations based on studies of reconstruction algorithms. The information included in this chapter will enable readers to create a simulation environment in which virtual tomography projections can be obtained in all basic projection systems. This monograph is a valuable study on computed tomography that will be of interest to advanced students and researchers in the fields of biomedical engineering, medical electronics, computer science and medicine.

Image reconstruction from projections. Probability and random variables. An overview of the process of CT. Physical problems associated with data collection in CT. Computer simulation of data collection in CT. Data collection and reconstruction of the head phantom under various assumptions. Basic concepts of reconstruction algorithms. Backprojection. Convolution method for parallel beams. Other transform methods for parallel beams. Convolution methods for divergent beams. The algebraic reconstruction techniques. Quadratic optimization methods. Noniterative series expansion methods. Truly three-dimensional reconstruction. Three-dimensional display of organs. Mathematical background.

Methods of the Fourier transform are widely used for practical applications of image reconstruction from projections, such as the computerized tomography. We mention the well known methods of back-projection and methods based on the Fourier slice theorem, which requires a crude interpolation when transforming the Fourier projections from the polar grid to the traditional Cartesian grid. The solution of this complex problem is very important in medical diagnoses, where projections data for reconstructing two- and three-dimensional images are obtained by means of the roentgen radiation with an investigated part of the body. In this work, we analyze solutions of the problem of reconstruction of the discrete image on the Cartesian grid from projections of the image on the spatial domain, which are based on the concept of the two-dimensional discrete tensor transformation. In the framework of the constructed model, we show a way of using the line-integrals of the image, or real projections data for exact reconstructing the discrete image. The model of image reconstruction proposed in this research is described for the cases, when the size of the Cartesian grid are primes and power of two. The problem we focus on is formulated as follows. For a given image $f(x, y)$ on the bounded region (such as the square $[0, 1] \times [0, 1]$) and the $N \times N$ Cartesian grid placed on the region, reconstruct exactly the discrete image $f_{n,m}$ from the line-integrals of the image $f(x, y)$ calculated in a finite number of projections. The solution of this problem is based on the new approach proposed by Grigoryan, which allows to transfer uniquely the geometry of the projections from the image plane to the geometry of projections onto the Cartesian grid. This transformation allows calculating the tensor representation of the discrete image, where the image is described by one-dimensional splitting-signals carrying the spectral information about the image at frequency-points of different subsets covering the Cartesian lattice. When the size of the image is a power of two, these subsets are intersected and this property can be used effectively for solution of the well known problem of image reconstruction from limited angle range projections. Our preliminary results show that the proposed method of reconstruction is more accurate than the known projections onto convex sets algorithm. In addition, the simulations of our algorithm demonstrate good reconstructions when the projections are within a limited angular range. The proposed method of image reconstruction is robust relative to the additive signal-independent noise in projection data.

This book contains a selection of communications presented at the Third International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, held 4-6 July 1995 at Domaine d' Aix-Marlioz, Aix-Ies-Bains, France. This nice resort provided an inspiring environment to hold discussions and presentations on new and developing issues. Roentgen discovered X-ray radiation in 1895 and Becquerel found natural radioactivity in 1896 : a hundred years later, this conference was focused on the applications of such radiations to explore the human body. If the physics is now fully understood, 3D imaging techniques based on ionising radiations are still progressing. These techniques include 3D Radiology, 3D X-ray Computed Tomography (3D-CT), Single Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET). Radiology is dedicated to morphological imaging, using transmitted radiations from an external X-ray source, and nuclear medicine to functional imaging, using radiations emitted from an internal radioactive tracer. In both cases, new 3D tomographic systems will tend to use 2D detectors in order to improve the radiation detection efficiency. Taking a set of 2D acquisitions around the patient, 3D acquisitions are obtained. Then, fully 3D image reconstruction algorithms are required to recover the 3D image of the body from these projection measurements.

This revised and updated second edition – now with two new chapters - is the only book to give a comprehensive overview of computer algorithms for image reconstruction. It covers the fundamentals of computerized tomography, including all the computational and mathematical procedures underlying data collection, image reconstruction and image display. Among the new topics covered are: spiral CT, fully 3D positron emission tomography, the linogram mode of backprojection, and state of the art 3D imaging results. It also includes two new chapters on comparative statistical evaluation of the 2D reconstruction algorithms and alternative approaches to image reconstruction.

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